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Interaction of Monopoles with Fermions in Higher Representations

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ABSTRACT

We show that for the fermions in the T = 3/2 representation, the Sathiapalan-Tomaras generalisation of their result for fermions in the T = 1 representation, cannot be obtained from the conservation laws of the full SU(2) theory. We explicitly construct the effective Lagrangian for this case, including the J = 0 and J = 1 gauge field fluctuations, and show that the equations of motion couple J = 0 and J = 1 fermions. But the Green's function involving couplings between the J = 0 and J = 1 fermions is suppressed.

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I. INTRODUCTION

The subject of monopole catalysis of baryon decay [1,2] has been of great interest in the past year. There have been several studies [3,4] of fermions in the isospinor representation of SU(2) interacting with the monopole, because this is the case of interest in the SU(5) GUT [5]. But, it is also of interest to study fermions in higher representations interacting with the monopole, which could be applicable in other GUTs. Such an analysis has been done for the T=1 case by Sathiapalan and Tomaras [6]. They have also argued that their results can be generalised to arbitrary representations T.

Another development in the study of the phenomenon of monopole catalysis has been made by Sen [4]. For the T=1/2 case he showed that the Rubakov-Callan effect is a consequence of the conservation laws of the full theory. Hence, though the boundary conditions at the monopole core were initially derived by solving the one-particle Dirac equation, they have a rigorous field theoretic justification, because the conservation laws of the full SU(2) theory lead uniquely to these boundary conditions.

In this paper, we try to rederive the Sathiapalan-Tomaras results using the conservation laws approach. For T = 1 fermions, we find that the conservation laws of the full SU(2) theory, along with the kinematic constraints do lead us to unique final states, in agreement with their results. But for $T=3/2$ fermions, the conservation laws allow certain processes, which are forbidden by the Sathiapalan-Tomaras analysis. This means that the effective Lagrangian obtained by solving the one-particle Dirac equation has more conservation laws than the original theory. Hence, their boundary conditions no longer have a field theoretic justification, and hence, merit a more careful analysis.

In section II, we use the conservation laws of Sen, along with the kinematic constraints that can be derived for the $T=1$ and $T=3/2$ cases, to predict final states, given the initial states. We show that for $T=3/2$ fermions, we do not have enough conditions to predict a unique final state for every initial state. In section III, we construct the effective Lagrangian for the $T=3/2$ fermions interacting with the monopole, allowing for both $J=0$ and $J=1$ gauge field excitations. In section IV, we show that in the absence of $J=1$ gauge field excitations, the effective Lagrangian satisfies more conservation laws than the full SU(2) theory. In the presence of these excitations however, the extra conservation laws are violated. But using the effective Lagrangian, we argue that the Sathiapalan-Tomaras forbidden processes are suppressed. In appendix A, we give some details of the derivation of the effective Lagrangian for the fermions interacting with the classical monopole field. In appendix B, we explain how the boundary conditions are derived in the absence of gauge field fluctuations.

II. CONSERVATION LAWS

In this section, we start with an SU(2) Lagrangian spontaneously broken to a U(1) by a triplet of Higgs. We also have massless fermions in an arbitrary representation of SU(2). The Lagrangian for such a system is

$$\mathcal{L} = -\frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + i \sum_{i=1}^{n} \bar{\psi}^i \not{D} \psi^i + (D_\mu \phi)^+ (D^\mu \phi) - V(\phi) \qquad 2.1$$

where ψ is a Dirac spinor, and n is the number of fermion families. We shall now analyse the system to find processes that are allowed by the conservation laws in the presence of the t'Hooft-Polyakov monopole

$$A_i^{cl} = \epsilon_{iaj} T_a \hat{r}_j \frac{F(r)}{r} \qquad 2.2$$

where F(r) vanishes at the origin, and tends to 1 exponentially outside the monopole core.

The conservation laws satisfied by this system have been analysed by Sen(4), and for the two-family case, he obtained the following conserved charges

$$S_1 = \overline{\psi_L^1} \gamma^0 \psi_L^1 + \overline{\psi_R^1} \gamma^0 \psi_R^1 \qquad 2.3$$

$$S_2 = \overline{\psi_L^2} \gamma^0 \psi_L^2 + \overline{\psi_R^2} \gamma^0 \psi_R^2 \qquad 2.4$$

$$S_3 = \overline{\psi_L^1} \gamma^0 \psi_L^1 - \overline{\psi_R^1} \gamma^0 \psi_R^1 - \overline{\psi_L^2} \gamma^0 \psi_L^2 + \overline{\psi_R^2} \gamma^0 \psi_R^2 \qquad 2.5$$

$$S_6 = \overline{\psi^1_L}\,\gamma^0\,\hat{r}\cdot\vec{T}\,\psi^1_L + \overline{\psi^1_R}\,\gamma^0\,\hat{r}\cdot\vec{T}\,\psi^1_R + \overline{\psi^2_L}\,\gamma^0\,\hat{r}\cdot\vec{T}\,\psi^2_L + \overline{\psi^2_R}\,\gamma^0\,\hat{r}\cdot\vec{T}\,\psi^2_R \qquad 2.6$$

S_1 and S_2 count the total number of particles of types 1 and 2, S_3 measures the differences in helicities of particles of types 1 and 2, and S_4 measures the total U(1) charge carried by the fermions. For the isospinor case (ψ = ($\chi_{1/2}$, $\chi_{-1/2}$)), he also showed that we obtain four more constraints by restricting the fermions to be in the J=0 partial wave so that $\hat{r}\cdot\vec{S} = -\hat{r}\cdot\vec{T}$. These eight constraints are sufficient to uniquely specify the final state for any initial state configuration. Hence, in this approach the conservation laws of the original SU(2) Lagrangian along with the kinematic helicity constraints uniquely lead us to baryon number violating processes.

What happens when we consider fermions in higher representations of SU(2) ? The conservation laws are still the same, but the kinematic constraints are now different.To see what the constraints are, let us reduce the relevant part of the Lagrangian ($=i\bar{\psi}\,\not{D}\,\psi$) in terms of two- component fields

$$\psi_L = \begin{pmatrix} \chi_L \\ -\chi_L \end{pmatrix} \quad ; \quad \psi_R = \begin{pmatrix} \chi_R \\ \chi_R \end{pmatrix} \qquad 2.7$$

Since for massless fields, the right and left handed fields decouple, we may write [6]

$$\mathcal{L} = \chi_R^+ \left[i\partial_0 + A_0 - \frac{1}{r} D_\Omega - i\vec{\sigma}\cdot\hat{r}\left(\partial_r + \frac{1}{r}\right)\right]\chi_R + \chi_L^+ \left[i\partial_0 + A_0 + \frac{1}{r} D_\Omega - i\vec{\sigma}\cdot\hat{r}\left(\partial_r + \frac{1}{r}\right)\right]\chi_L \qquad 2.8$$

where $D_\Omega = D_{\Omega_0} + \Delta$.

$$D_{\Omega_0} = i r \sigma_i \left(\delta_{ij} - \hat{r}_i \hat{r}_j\right)\partial_j - \sigma_i \epsilon_{iaj} T_a \hat{r}_j - i\hat{r}\cdot\vec{\sigma} \qquad 2.9$$

$$\Delta = \sigma_i \epsilon_{iaj} T_a \hat{r}_j \qquad 2.10$$

We can easily verify that D_{Ω_0} translates into an angular momentum barrier of the form $\{J^2+1/4-(\hat{r}\cdot\vec{T})^2\}^{1/2}$. This specifies the possible J value for each $\hat{r}\cdot\vec{T}$ value.

(A) Fermions in the T=1 Representation.

$$\psi = \begin{pmatrix} \chi_1 \\ \chi_0 \\ \chi_{-1} \end{pmatrix} \qquad 2.11$$

The subscript on χ labels its $\hat{r}\cdot\vec{T}$ value. It is clear that only the J=1/2 partial wave of χ_1 and χ_{-1} can enter the monopole core, whereas χ_0 is prevented from entering the monopole core for any value of J. Since $\hat{r}\cdot\vec{J} = \hat{r}\cdot\vec{T} + \hat{r}\cdot\vec{S}$, we have χ_{1R} and χ_{-1L} as incoming fermions and χ_{1L} and χ_{-1R} as outgoing fermions. χ_0 decouples from the problem, and effectively (χ_1, χ_{-1}) acts as a doublet. So here again the conservation laws along with the physical angular momentum constraints yield the right number of conditions to lead to a unique final state for every initial state. For excample, we have

$$\chi_{1R}^{1} + \chi_{-1L}^{2} \longrightarrow \chi_{1L}^{2} + \chi_{-1R}^{1}$$

uniquely.

(B) Fermions in the T=3/2 Representation

$$\psi = \begin{pmatrix} \chi_{3/2} \\ \chi_{1/2} \\ \chi_{-1/2} \\ \chi_{-3/2} \end{pmatrix} \qquad 2.13$$

As before, the subscript on χ labels its $\hat{r}\cdot\vec{T}$ value. The angular momentum barrier term allows only the J=1 partial wave for the $\hat{r}\cdot\vec{T}=\pm 3/2$ fermions and the J=0 partial wave for the $\hat{r}\cdot\vec{T}=\pm 1/2$ fermions to enter the monopole core. $\hat{r}\cdot\vec{J} = \hat{r}\cdot\vec{T} + \hat{r}\cdot\vec{S}$ further tells us that $\chi_{3/2R}$, $\chi_{1/2R}$, $\chi_{-1/2L}$, and $\chi_{-3/2L}$ are incoming fermions and $\chi_{3/2L}$, $\chi_{1/2L}$, $\chi_{-1/2R}$ and $\chi_{-3/2R}$ are outgoing fermions. But now the constraints that we have are not sufficient to uniquely specify a final state for every initial state.For example,

$$\chi^{1}_{3/2R} + \chi^{2}_{-3/2L} \longrightarrow \chi^{2}_{3/2L} + \chi^{1}_{-3/2R} \qquad 2.14$$

$$\longrightarrow \chi^{2}_{1/2L} + \chi^{1}_{-1/2R} \qquad 2.15$$

Both final states are allowed by the conservation laws and the kinematic constraints,though 2.15 is not allowed by the Sathiapalan-Tomaras extension of their result for T=1. In their case, ($\chi_{3/2}$, $\chi_{-3/2}$) and ($\chi_{1/2}$, $\chi_{-1/2}$) act as decoupled doublets, and 2.14 is the only final state. As explained in the introduction, this discrepancy warrants a more careful investigation of the problem.

III. EFFECTIVE LAGRANGIAN FOR ISOSPIN 3/2 FERMIONS

In this section, we shall reduce the fermion and gauge terms in the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\, \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + i \sum_{i=1}^{n} \bar{\psi}^i \not{D} \psi^i + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) \qquad 3.1$$

to a more tractable form in terms of the dynamical variables of the problem.

(A) Fermion-A^{cl} Interaction

Since we are dealing with massless fields, the left and right handed components decouple, and hence, for convenience, we shall derive the effective Lagrangian in terms of the right handed fields.

$$\mathcal{L} = \bar{\chi}_R \left(i\partial_0 + A_0 - i\vec{\sigma}\cdot\hat{r}\left(\partial_r + \frac{1}{r}\right) - \frac{D\Omega}{r}\right) \chi_R \qquad 3\text{-}2$$

We expand the fermion fields in J=0 and J=1 partial waves using the basis in which J^2, J_3, $\hat{r}\cdot\vec{S}$ and $\hat{r}\cdot\vec{T}$ are diagonal

$$\chi_R^{J=0}(\vec{r},t) = L_{\frac{1}{2},-\frac{1}{2}}(\theta,\phi)\, \tilde{U}^{J=0}_{\frac{1}{2},-\frac{1}{2}R}(r,t) + L_{-\frac{1}{2},\frac{1}{2}}(\theta,\phi)\, \tilde{U}^{J=0}_{-\frac{1}{2},\frac{1}{2}R}(r,t) \qquad 3\cdot 3$$

$$\chi_R^{J=1}(\vec{r},t) = M^m_{\frac{1}{2},-\frac{1}{2}}(\theta,\phi)\, \tilde{U}^m_{\frac{1}{2},-\frac{1}{2}}(r,t) + M^m_{-\frac{1}{2},\frac{1}{2}}(\theta,\phi)\, \tilde{U}^m_{-\frac{1}{2},\frac{1}{2}}(r,t)$$

$$+ \; M^{m}_{1/2,1/2}(\theta,\phi)\,\tilde{U}^{m}_{1/2,1/2}(r,t) \;+\; M^{m}_{-1/2,-1/2}(\theta,\phi)\,\tilde{U}^{m}_{-1/2,-1/2}(r,t)$$

$$+ \; M^{m}_{3/2,-1/2}(\theta,\phi)\,\tilde{U}^{m}_{3/2,-1/2}(r,t) \;+\; M^{m}_{-3/2,1/2}(\theta,\phi)\,\tilde{U}^{m}_{-3/2,1/2}(r,t) \qquad 3.4$$

$L_{1/2,-1/2}$ and $L_{-1/2,1/2}$ represent J=0 tensors with eigenvalues $L_{\hat{r}\cdot\vec{T},\hat{r}\cdot\vec{S}}$. Since J=0 $\Longrightarrow$ $J_3 = 0$, we do not indicate it. $M^{m}_{1/2,-1/2}$, $M^{m}_{-1/2,1/2}$, $M^{m}_{-1/2,-1/2}$, $M^{m}_{1/2,1/2}$, $M^{m}_{3/1,-1/2}$, and $M^{m}_{-3/2,1/2}$ are J=1 tensors with eigen values $M^{J_3}_{\hat{r}\cdot\vec{T},\hat{r}\cdot\vec{S}}$. The explicit tensor structures have been given in Appendix A.The same eigenvalue notation has been used for the $\tilde{U}$ variables.

The details of the derivation of the effective Lagrangian from 3.1 using explicit tensor structures have been outlined in Appendix A. Here, we shall only mention the following points. We work in the $A_0 = 0$ gauge. We make explicit a 1/r dependence of the U variables (i.e. U = $\tilde{U}/r$), so that we may replace (∂r + 1/r) by ∂r. The results after angular integration are given below.

$$L_{f_R\text{-}A^{cl}} = \int \mathcal{L}_{f_R\text{-}A^{cl}} \frac{d\Omega}{4\pi}$$

$$= \frac{1}{r^2}\Big[\; U^{+m}_{3/2,-1/2R}\Big\{ (i\partial_0 + i\partial_r)\, U^{m}_{3/2,-1/2R} - \frac{\sqrt{3}}{r}(1-F(r))\, U^{m}_{1/2,1/2R}\Big\}$$

$$+ \; U^{+m}_{-3/2,1/2R}\Big\{ (i\partial_0 - i\partial_r)\, U^{m}_{-3/2,1/2R} - \frac{\sqrt{3}}{r}(1-F(r))\, U^{m}_{-1/2,-1/2R}\Big\}$$

$$+ \; U^{+m}_{1/2,1/2R}\Big\{ (i\partial_0 - i\partial_r)\, U^{m}_{1/2,1/2R} - \sqrt{\frac{3}{r}}\,(1-F(r))\, U^{m}_{3/2,-1/2R}$$

$$+ \frac{\sqrt{2}}{r} U^{m}_{1/2,-1/2R} \Big\}$$

$$+ U^{+m}_{-1/2,-1/2R} \Big\{ (i\partial_0 + i\partial_r) U^{m}_{-1/2,-1/2R} - \frac{\sqrt{3}}{r} (1-F(r)) U^{m}_{-3/2,1/2R} + \frac{\sqrt{2}}{r} U^{m}_{-1/2,1/2R} \Big\}$$

$$+ U^{+m}_{1/2,-1/2R} \Big\{ (i\partial_0 + i\partial_r) U^{m}_{1/2,-1/2R} - \frac{2i}{r} (1-F(r)) U^{m}_{-1/2,1/2R} + \frac{\sqrt{2}}{r} U^{m}_{1/2,1/2R} \Big\}$$

$$+ U^{+m}_{-1/2,1/2R} \Big\{ (i\partial_0 - i\partial_r) U^{m}_{-1/2,1/2R} + \frac{2i}{r} (1-F(r)) U^{m}_{1/2,-1/2R} + \frac{\sqrt{2}}{r} U^{m}_{-1/2,-1/2R} \Big\}$$

$$+ U^{+J=0}_{1/2,-1/2R} \Big\{ (i\partial_0 + i\partial_r) U^{J=0}_{1/2,-1/2R} - \frac{2i}{r} (1-F(r)) U^{J=0}_{-1/2,1/2R} \Big\}$$

$$+ U^{+J=0}_{-1/2,1/2R} \Big\{ (i\partial_0 - i\partial_r) U^{J=0}_{-1/2,1/2R} + \frac{2i}{r} (1-F(r)) U^{J=0}_{1/2,-1/2R} \Big\} \Big] \qquad 3.5$$

As we had expected, in the absence of gauge field fluctuations there is no term in the Lagrangian coupling the J=0 and J=1 fermions.The boundary conditions at the monopole core that the fermion fields have to satisfy, have been derived in Appendix B, and in the limit $r_0 \to 0$,we get

$$U^{m}_{3/2,-1/2R} + U^{m}_{-3/2,1/2R} \longrightarrow 0 \qquad 3.6$$

$$U^{m}_{1/2,1/2R} \longrightarrow 0 \qquad 3.7$$

$$U^{m}_{-1/2,-1/2R} \longrightarrow 0 \qquad 3.8$$

$$U^{m}_{1/2,-1/2R} \longrightarrow 0 \qquad 3.9$$

$$U^{m}_{-1/2,1/2R} \longrightarrow 0 \qquad 3.10$$

$$U^{J=0}_{1/2,-1/2R} + U^{J=0}_{-1/2,1/2R} \longrightarrow 0 \qquad 3.11$$

The boundary conditions imply that ($U_{3/2,-1/2R}, U_{-3/2,1/2R}$) and $U_{1/2,-1/2R}, U_{-1/2,1/2R}$) act as decoupled doublets, while the other four components decouple from the problem. Exactly as in the Rubakov-Callan case, we may compute Green's functions involving the two decoupled doublets and we get non-zero Green's functions only for the Sathiapalan-Tomaras allowed processes.

(B) Classical and Fluctuating Gauge Interactions

In the Rubakov-Callan case, only the J=0 fluctuations were included, since for the isospinor case, only the J=0 partial wave of the fermions could enter the monopole core. But here, both J=0 and J=1 partial waves are allowed near the monopole core, and so , we have no rationale for ignoring the J=1 fluctuations. In fact, if there exist non-zero couplings between the J=0 and J=1 sectors, we expect it to occur through such excitations. We may expand

$$A_{ia} = A_{ia}^{cl} + (\ J=0 \ \text{fluctuations}) + (\ J=1 \ \text{fluctuations}) \qquad 3.12$$

By using the explicit tensor structures of the fluctuating fields and computing their mass and kinetic energy terms, we may try to look for the relevant fluctuations, which are not suppressed in the effective action.But a simpler way is to use Callan's trick to find out those excitations that are massless and have no contribution from the $F_{ij}^{a}\ F^{ija}$ term. We use

$$A_i^{\lambda} = U_\lambda A_i U_\lambda^{-1} + i\, U_\lambda \partial_i U_\lambda^{-1} \qquad 3.13$$

where $U_\lambda = e^{i\lambda(\vec{r},t)\,\hat{r}\cdot\vec{T}}$ and $A_i = A_{ia} T_a$.
When λ does not have any time dependence, this is merely a gauge transformation and hence costs no energy.But the time dependence of λ leads to a nonzero F_{0i} of the form

$$\begin{aligned} F_{0i} &= \partial_0 A_i^{\lambda} \\ &= \frac{\dot{\lambda}}{r}\left(\delta_{ij} - \hat{r}_i\hat{r}_j\right)\left(1-F(r)\right) e^{i\lambda(\vec{r},t)\hat{r}\cdot T}\, T_j\, e^{-i\lambda(\vec{r},t)\hat{r}\cdot T} \\ &\quad + \left(\partial_i\dot{\lambda}\right)\hat{r}\cdot\vec{T} \end{aligned} \qquad 3.14$$

(As mentioned earlier, we are working in the $A_0 = 0$ gauge.) To seperate excitations into J=0 and J=1 parts, we make a partial wave expansion of $\lambda(r,t)$ keeping only terms upto L=1.

$$\lambda(\vec{r},t) = \lambda_{00}(r,t) + \lambda_{1m}(r,t)\,\hat{r}_m \qquad 3.15$$

We use this in 3.14 to compute

$$\mathrm{Tr}\ F_{0i}F^{0i} = \frac{1}{2}\left(\dot{\lambda}_{00}' + \dot{\lambda}_{im}'\cdot\hat{r}_m\right)^2 + \frac{1}{2}\sum_i\left[\frac{\dot{\lambda}_{im}}{r}\left(\delta_{im} - \hat{r}_i\hat{r}_m\right)\right]^2 + \frac{\left(\dot{\lambda}_{00} + \dot{\lambda}_{im}\cdot\hat{r}_m\right)^2}{r^2}\left(1-F(r)\right)^2 \qquad 3.16$$

Hence, the term in the effective action is (after angular integration)

$$L_{gauge} = \int \mathcal{L}_g \frac{d\Omega}{4\pi} = L_{A^{cl}} + \frac{1}{2}\dot{\lambda}_{oc}'^{\,2} + \frac{1}{6}\dot{\lambda}_{im}'^{\,2} + \frac{1}{3}\dot{\lambda}_{im}^2/r^2 + \left(\frac{\dot{\lambda}_{oo}^2 + \dot{\lambda}_{im}^2/3}{r^2}\right)\left(1-F(r)\right)^2$$

$$= L_{A^{cl}} + L_{\lambda_{oo}} + L_{\lambda_{im}} \qquad 3.17$$

(C) Fermion-Fluctuation Interaction

We are interested in the effective Lagrangian from the term $\bar{\psi}\ \not{A}_{f1}\ \psi$. Let us make a change of variables to get it in a simpler form

$$\psi \rightarrow \psi' = e^{i\lambda(\vec{r},t)\,\hat{r}\cdot\vec{T}}\,\psi \qquad 3.18$$

As explained in the previous sub-section, this is almost a gauge transformation. But the time dependence of λ introduces the following terms in the effective interaction (after angular integration.)

$$L_{f_R'-\lambda} = \int \mathcal{L}_{f_R'-\lambda}\frac{d\Omega}{4\pi}$$

$$= \frac{\dot{\lambda}_{oo}}{r^2}\Big[\frac{1}{2}U'^{+m}_{\frac12,-\frac12 R}U'^{m}_{\frac12,-\frac12 R} - \frac{1}{2}U'^{+m}_{-\frac12,\frac12 R}U'^{m}_{-\frac12,\frac12 R}$$

$$+\frac{1}{2}U'^{+m}_{\frac12,\frac12 R}U'^{m}_{\frac12,\frac12 R} - \frac{1}{2}U'^{+m}_{-\frac12,-\frac12 R}U'^{m}_{-\frac12,-\frac12 R}$$

$$+\frac{3}{2}U'^{+m}_{\frac32,-\frac12 R}U'^{m}_{\frac32,-\frac12 R} - \frac{3}{2}U'^{+m}_{-\frac32,\frac12 R}U'^{m}_{-\frac32,\frac12 R}$$

$$+\frac{1}{2}U'^{+J=0}_{\frac12,-\frac12 R}U'^{J=0}_{\frac12,-\frac12 R} - \frac{1}{2}U'^{+J=0}_{-\frac12,\frac12 R}U'^{J=0}_{-\frac12,\frac12 R}\Big]$$

$$+\left\{\frac{\dot{\lambda}_{1m}}{\pi^2}\left[\frac{1}{2}\, U'^{+m}_{1/2,-1/2R}\, U'^{J=0}_{1/2,-1/2R} - \frac{1}{2}\, U'^{+m}_{-1/2,1/2R}\, U'^{J=0}_{-1/2,1/2R}\right] + h.c.\right\}$$

$$= L_{f'_R-\lambda_{00}} + L_{f'_R-\lambda_{1m}} \qquad 3.19$$

where the separation into $L_{f'_R\lambda_{00}}$ and $L_{f'_R-\lambda_{1m}}$ is obvious. The point to note is that terms in $L_{f'_R-\lambda_{1m}}$ couple J=0 fermions to J=1 fermions.The prime on the fermion fields indicate that they are the transformed fields. The full effective Lagrangian can now be written in terms of the transformed fields.Incorporating the left and right handed fields, the full effective action is

$$\begin{aligned} L_{eff} = {} & L_{A^{cl}} + L_{f_R'-A^{cl}} + L_{f_L'-A^{cl}} \\ & + L_{\lambda_{00}} + L_{f_R'-\lambda_{00}} + L_{f_L'-\lambda_{00}} \\ & + L_{\lambda_{1m}} + L_{f_R'-\lambda_{1m}} + L_{f_L'-\lambda_{1m}} \end{aligned} \qquad 3.20$$

We shall henceforth drop the primes on the fermion fields.

IV. DISCUSSION AND CONCLUSION

In this section, we shall look at the conservation laws of the effective Lagrangian and see how they are affected by the J = 1 gauge field fluctuations. We shall see that J = 1 excitations do couple J = 0 fermions to J = 1 fermions, although processes involving such couplings may be suppressed.

As in section II, our theory contains two T = 3/2 fermion representations interacting with the monopole. In the absence of λ_{1m} terms, the effective Lagrangian for the system is

$$L_{eff} = L_{A^{cl}} + \sum_{i=1}^{2} L_{f_L^i - A^{cl}} + \sum_{i=1}^{2} L_{f_R^i - A^{cl}} + L_{\lambda_{00}} + \sum_{i=1}^{2} L_{f_L^i - \lambda_{00}} + \sum_{i=1}^{2} L_{f_R^i - \lambda_{00}} \qquad 4.1$$

This Lagrangian is symmetric under the following transformations,

$$\psi_L^i \longrightarrow \psi_L^{i\prime} = e^{i\theta_{iL}} \psi_L^i \qquad 4.2$$

$$\psi_R^i \longrightarrow \psi_R^{i\prime} = e^{i\theta_{iR}} \psi_R^i \qquad 4.3$$

$$\chi_L^i \longrightarrow \chi_L^{i\prime} = e^{i\phi_{iL}} \chi_L^i \qquad 4.4$$

$$\chi_R^i \longrightarrow \chi_R^{i\prime} = e^{i\phi_{iR}} \chi_{iR} \qquad 4.5$$

where ψ^i denotes the set ($U^{mi}_{3/2,-1/2}$, $U^{mi}_{-3/2,1/2}$, $U^{mi}_{1/2,1/2}$, $U^{mi}_{-1/2,-1/2}$, $U^{mi}_{1/2,-1/2}$ and $U^{mi}_{-1/2,+1/2}$) and χ^i denotes the set $\left(U^{J=0,i}_{1/2,-1/2} \, , \, U^{J=0,i}_{-1/2,1/2} \right)$. Since one of the charges associated with the above symmetries is anomalous, along with the local U (1) charge, we have the following eight conserved charges

$$S_i^{J=0} = \int dr \left(\chi_R^{i+} \chi_R^i + \chi_L^{i+} \chi_L^i \right) \tag{4.6}$$

$$S_i^{J=1} = \int dr \left(\psi_R^{i+} \psi_R^i + \psi_L^{i+} \psi_L^i \right) \tag{4.7}$$

$$S_3^{J=0} = \int dr \left(\chi_R^{1+} \chi_R^1 - \chi_L^{1+} \chi_L^1 - \chi_R^{2+} \chi_R^2 + \chi_L^{2+} \chi_L^2 \right) \tag{4.8}$$

$$S_3^{J=1} = \int dr \left(\psi_R^{1+} \psi_R^1 - \psi_L^{1+} \psi_L^1 - \psi_R^{2+} \psi_R^2 + \psi_L^{2+} \psi_L^2 \right) \tag{4.9}$$

$$S_3' = \int dr \left(\psi_R^{1+} \psi_R^1 - \psi_L^{1+} \psi_L^1 + \psi_R^{2+} \psi_R^2 - \psi_L^{2+} \psi_L^2 \right) - 11 \left(\chi_R^{1+} \chi_R^1 - \chi_L^{1+} \chi_L^1 + \chi_R^{2+} \chi_R^2 - \chi_L^{2+} \chi_L^2 \right) \tag{4.10}$$

$$
\begin{aligned}
S_4 = \sum_{i=1}^{2} \int dr \Big\{ & \frac{3}{2} \Big(U^{+mi}_{3/2,-1/2}\, U^{mi}_{3/2,-1/2} - U^{+mi}_{-3/2,1/2}\, U^{mi}_{-3/2,1/2} \Big) \\
& + \frac{1}{2} \Big(U^{+mi}_{1/2,1/2}\, U^{mi}_{1/2,1/2} - U^{+mi}_{-1/2,-1/2}\, U^{mi}_{-1/2,-1/2} \Big) \\
& + \frac{1}{2} \Big(U^{+mi}_{1/2,-1/2}\, U^{mi}_{1/2,-1/2} - U^{+mi}_{-1/2,1/2}\, U^{mi}_{-1/2,1/2} \Big) \\
& + \frac{1}{2} \Big(U^{+J=0i}_{1/2,-1/2}\, U^{J=0i}_{1/2,-1/2} - U^{+J=0i}_{-1/2,1/2}\, U^{J=0i}_{-1/2,1/2} \Big) \Big\}
\end{aligned}
\qquad 4.11
$$

Conservation of these charges lead to a unique final state for any given initial state and is sufficient to rule out process 2.15. But we know that the original SU(2) theory had only four conservation laws. In fact, the correspondence between the original conserved quantites defined in equations 2.3 through 2.6 is

$$S_1^{J=0} + S_1^{J=1} = S_1 \qquad 4.12$$

$$S_2^{J=0} + S_2^{J=1} = S_2 \qquad 4.13$$

$$S_3^{J=0} + S_3^{J=1} = S_3 \qquad 4.14$$

$$S_4 = S_4 \qquad 4.15$$

Since the extra conservation laws do not come from the full theory, they need not be stable under further gauge field fluctuations. In fact, if we introduce λ_{1m} terms, we can no longer make independent transformations on the J = 0 and J = 1 fermions - i.e., we are restricted to transformations with $\theta_{iL} = \phi_{iL}$ and $\theta_{iR} = \phi_{iR}$ in equations 4.2 through 4.5. Hence the full effective Lagrangian, equation 3.20, conserves only the charges S_1 through S_4 and should allow processes of the form 2.15. Unfortunately, the conservation laws do not give any information about the amplitudes of the various allowed processes. To see whether any given process is suppressed or enhanced, we have to compute its Green's function explicitly. For the process 2.15, we are not able to calculate the Green's function exactly, but let us see how far we can go with the information that we do have.

The equations of motion with $\dot{\lambda}_{1m} = 0$ are (in terms of the variables A,B,C,... defined in Appendix A.)

$$\partial_r A + \frac{\sqrt{3}}{r} i \left(1 - F(r)\right) D = 0 \qquad 4.16$$

$$-\partial_r D + \frac{\sqrt{3}}{r} i \left(1 - F(r)\right) D - \frac{\sqrt{2}\, i E}{r} = 0 \qquad 4.17$$

$$\partial_r E - \frac{\sqrt{2}}{r} i D + \frac{2}{r} \left(1 - F(r)\right) E - i \dot{\lambda}_{1m} H = 0 \qquad 4.18$$

$$\partial_r H - \frac{2}{r}\left(1-F(r)\right)H - i\dot{\lambda}_{lm} E = 0 \qquad 4.19$$

$$\partial_r B + \frac{\sqrt{3}\,i}{r}\left(1-F(r)\right)C = 0 \qquad 4.20$$

$$-\partial_r C + \frac{\sqrt{3}\,i}{r}\left(1-F(r)\right)B - \frac{\sqrt{2}}{r}\,i K = 0 \qquad 4.21$$

$$\partial_r K - \frac{\sqrt{2}\,i}{r}C - \frac{2}{r}\left(1-F(r)\right)K - i\dot{\lambda}_{lm} G = 0 \qquad 4.22$$

$$\partial_r G + \frac{2}{r}\left(1-F(r)\right)G - i\dot{\lambda}_{lm} K = 0 \qquad 4.23$$

To affect the boundary conditions near the origin, it is clear that $\dot{\lambda}_{1m}$ should have a $1/r$ dependence - i.e. $\dot{\lambda}_{1m} = a/r \times \hat{r}_m$. We could use this as an ansatz and derive the boundary conditions at the origin. Equations 4.7 through 4.10 form one set and equations 4.11 through 4.14 form another set, and in general, it is clear that we shall get boundary conditions coupling J=0 fermions to J=1 fermions. But when we use this ansatz in the effective action

$$S_{eff} = \int L\, r^2 dr\, dt \qquad 4.24$$

we get terms of the form

$$a^2 \int\!\!\int_{r_0}^{\infty} \frac{1}{r^4} r^2 dr\, dt \simeq a^2 \int \frac{1}{r_0}\, dt \qquad 4.25$$

Hence the effective action blows up as $1/r_0$, causing a suppression, unless the excitation lasts only for a time of order r_0, in which case, it is unlikely that these momentary boundary conditions lead to any new processes. If we take any softer dependence of $\dot{\lambda}_{1m}$ on r (e.g., $\dot{\lambda}_{1m}$ = alnr leads to a finite action), the boundary conditions at the origin are not affected, and we get back the same boundary conditions that we derived in Appendix B. Hence, we conclude that the process

$$\chi_{3/2} + \chi_{-3/2} \rightarrow \chi_{1/2} + \chi_{-1/2}$$

is suppressed.

We should emphasize the assumptions that go into deriving the above result. We have assumed that a momentary boundary condition coupling the J=0 fermions to J=1 fermions cannot yield any new process. This is a plausible assumption, but we cannot prove it. We have also considered only massless modes of excitation. This again seems reasonable, since we would expect massive excitations to have an r_0 suppression, but it has not been rigorously proved.

Hence, our conclusion, with the caveats mentioned above is that the process

$$\chi_{3/2} + \chi_{-3/2} \rightarrow \chi_{1/2} + \chi_{-1/2}$$

is suppressed, even though there is no conservation law in the full SU(2) theory to prevent such a process.

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APPENDIX A

Here, we would like to outline the steps leading to the effective Lagrangian for the fermionic sector. As we had mentioned in Section II, we work in the J, $J_3 = m$, $\hat{r}\cdot\vec{T}$, $\hat{r}\cdot\vec{S}$ diagonal basis.

$$X_R^{J=0} = L_{1/2,-1/2}\, U^{J=0}_{1/2,-1/2\,R} + L_{-1/2,1/2}\, U^{J=0}_{-1/2,1/2\,R} \qquad A1$$

$$\begin{aligned} X_R^{J=1} = {} & M^m_{1/2,-1/2}\, U^m_{1/2,-1/2\,R} + M^m_{-1/2,+1/2}\, U^m_{-1/2,1/2\,R} \\ & + M^m_{1/2,1/2}\, U^m_{1/2,1/2\,R} + M^m_{-1/2,-1/2}\, U^m_{-1/2,1/2\,R} \\ & + M^m_{3/2,1/2}\, U^m_{3/2,-1/2\,R} + M^m_{-3/2,1/2}\, U^m_{-3/2,1/2\,R} \end{aligned} \qquad A2$$

Let me define the following linearly independent tensors

$$\begin{aligned} P^1_{ijk\alpha} = {} & (\hat{r}\cdot\vec{\tau}\epsilon)_{ij}\,\epsilon_{k\alpha} + (\hat{r}\cdot\vec{\tau}\epsilon)_{jk}\,\epsilon_{i\alpha} \\ & + (\hat{r}\cdot\vec{\tau}\epsilon)_{ik}\,\epsilon_{j\alpha} \end{aligned} \qquad A3$$

$$\begin{aligned} P^2_{ijk\alpha} = {} & (\hat{r}\cdot\vec{\tau}\epsilon)_{ij}(\hat{r}\cdot\vec{\tau}\epsilon)_{k\alpha} + (\hat{r}\cdot\vec{\tau}\epsilon)_{jk}(\hat{r}\cdot\vec{\tau}\epsilon)_{i\alpha} \\ & + (\hat{r}\cdot\vec{\tau}\epsilon)_{ik}(\hat{r}\cdot\vec{\tau}\epsilon)_{j\alpha} \end{aligned} \qquad A3$$

$$\begin{aligned} Q^1_{ijk\alpha n_1 n_2} = {} & \epsilon_{k\alpha}(\tau_p\epsilon)_{ij}(\tau_p\epsilon)_{n_1 n_2} + {} \\ & \epsilon_{i\alpha}(\tau_p\epsilon)_{jk}(\tau_p\epsilon)_{n_1 n_2} + \epsilon_{j\alpha}(\tau_p\epsilon)_{ik}(\tau_p\epsilon)_{n_1 n_2} \end{aligned} \qquad A4$$

$$
\begin{aligned}
Q^2_{ijk\alpha n_1 n_2} = \; & (\hat{\gamma}\cdot\vec{\tau}\epsilon)_{k\alpha}\,(\tau_\rho\epsilon)_{ij}\,(\tau_\rho\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\cdot\vec{\tau}\epsilon)_{i\alpha}\,(\tau_\rho\epsilon)_{jk}\,(\tau_\rho\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\,\vec{\tau}\epsilon)_{j\alpha}\,(\tau_\rho\epsilon)_{ik}\,(\tau_\rho\epsilon)_{n_1 n_2}
\end{aligned}
\qquad \text{A 6}
$$

$$
\begin{aligned}
Q^3_{ijk\alpha n_1 n_2} = \; & (\hat{\gamma}\,\vec{\tau}\epsilon)_{ij}\,(\tau_\rho\epsilon)_{k\alpha}\,(\tau_\rho\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\cdot\tau\epsilon)_{jk}\,(\tau_\rho\epsilon)_{i\alpha}\,(\tau_\rho\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\cdot\tau\epsilon)_{ik}\,(\tau_\rho\epsilon)_{j\alpha}\,(\tau_\rho\epsilon)_{n_1 n_2}
\end{aligned}
\qquad \text{A 7}
$$

$$
\begin{aligned}
Q^4_{ijk\alpha n_1 n_2} = \; & (\hat{\gamma}\cdot\vec{\tau}\epsilon)_{k\alpha}\,\epsilon_{abc}\,\hat{r}_a\,(\tau_b\epsilon)_{ij}\,(\tau_c\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\cdot\tau\epsilon)_{i\alpha}\,\epsilon_{abc}\,\hat{r}_a\,(\tau_b\epsilon)_{jk}\,(\tau_c\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\cdot\tau\epsilon)_{j\alpha}\,\epsilon_{abc}\,\hat{r}_a\,(\tau_b\epsilon)_{ik}\,(\tau_c\epsilon)_{n_1 n_2}
\end{aligned}
\qquad \text{A 8}
$$

$$
\begin{aligned}
Q^5_{ijk\alpha n_1 n_2} = \; & \epsilon_{k\alpha}\,(\hat{\gamma}\cdot\tau\epsilon)_{ij}\,(\hat{\gamma}\cdot\tau\epsilon)_{n_1 n_2} \\
& + \epsilon_{i\alpha}\,(\hat{\gamma}\cdot\tau\epsilon)_{jk}\,(\hat{\gamma}\cdot\tau\epsilon)_{n_1 n_2} \\
& + \epsilon_{j\alpha}\,(\hat{\gamma}\cdot\tau\epsilon)_{ik}\,(\hat{\gamma}\cdot\tau\epsilon)_{n_1 n_2}
\end{aligned}
\qquad \text{A 9}
$$

$$
\begin{aligned}
Q^6_{ijk\alpha n_1 n_2} = \; & (\hat{\gamma}\cdot\tau\epsilon)_{k\alpha}\,(\hat{\gamma}\cdot\tau\epsilon)_{ij}\,(\hat{\gamma}\cdot\tau\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\,\tau\epsilon)_{i\alpha}\,(\hat{\gamma}\cdot\tau\epsilon)_{ij}\,(\hat{\gamma}\cdot\tau\epsilon)_{n_1 n_2} \\
& + (\hat{\gamma}\cdot\tau\epsilon)_{j\alpha}\,(\hat{\gamma}\cdot\tau\epsilon)_{ik}\,(\hat{\gamma}\cdot\tau\epsilon)_{n_1 n_2}
\end{aligned}
\qquad \text{A 10}
$$

Here i,j,k = 1,2 are isospin indices, α = 1,2 is a spin index. n_1 and n_2 are indices which always appear symmetrically and it can be identified with the J_3 index as follows

$$(\hat{\gamma}\cdot\tau\epsilon)_{11} = -\hat{\gamma}^{1} \qquad \text{A 11}$$

$$(\hat{\gamma}\cdot\tau\epsilon)_{22} = \hat{\gamma}^{-1} \qquad \text{A 1}$$

$$(\hat{\gamma}\cdot\tau\epsilon)_{12+21} = \hat{\gamma}^{0} \qquad \text{A 13}$$

Next, we look at the linear combinations of these tensors that are eigen functions of $\hat{r}\cdot\vec{T}$ and $\hat{r}\cdot\vec{S}$ and hence identify

$$L_{1/2,-1/2} = (P^1 + P^2)\,/\,4\sqrt{3} \qquad \text{A 14}$$

$$L_{-1/2,1/2} = (P^1 - P^2)\,/\,4\sqrt{3} \qquad \text{A 15}$$

$$M^m_{1/2,-1/2} = (Q^5 + Q^6)\,/\,4\sqrt{3} \qquad \text{A 16}$$

$$M^m_{-1/2,1/2} = (Q^5 - Q_6)\,/\,4\sqrt{3} \qquad \text{A 17}$$

$$M^m_{3/2,-1/2} = \frac{(Q^1 + 2Q^2 - Q^3 - iQ^4 - Q^5 - Q^6)}{12\sqrt{2}} \qquad \text{A 18}$$

$$M^m_{-3/2,1/2} = \frac{(Q^1 - 2Q^2 + Q^3 - iQ^4 - Q^5 + Q^6)}{12\sqrt{2}} \qquad \text{A 19}$$

$$M^m_{1/2,1/2} = \frac{(-iQ^1 + iQ^3 + Q^4 + iQ^5 - iQ^6)}{4\sqrt{6}} \qquad \text{A 20}$$

$$M^m_{-1/2,-1/2} = \frac{(iQ^1 + iQ^3 - Q^4 - iQ^5 - iQ^6)}{4\sqrt{6}} \qquad \text{A 21}$$

The normalisation factor comes from requiring $\int L^*_{1/2,-1/2}\, L_{1/2,-1/2}\, \frac{d\Omega}{4\pi} = 1$ and similarly for the rest.

To calculate the effective Lagrangian, we need the effect of D_{Ω_0} and Δ on these tensors. (D_{Ω_0} and Δ have been defined in equations 2.9 and 2.10). The results are the following

$$D_{\Omega_0}\, L_{1/2,-1/2} = 0 \qquad \text{A 22}$$

$$D_{\Omega_0}\, L_{-1/2,1/2} = 0 \qquad \text{A 23}$$

$$D_{\Omega_0}\, M_{1/2,-1/2} = -\sqrt{2}\, M_{1/2,1/2} \qquad \text{A 24}$$

$$D\Omega_0 \, M_{-1/2,1/2} = -\sqrt{2} \, M_{-1/2,-1/2} \tag{A25}$$

$$D\Omega_0 \, M_{1/2,1/2} = -\sqrt{2} \, M_{1/2,-1/2} \tag{A26}$$

$$D\Omega_0 \, M_{-1/2,-1/2} = -\sqrt{2} \, M_{-1/2,-1/2} \tag{A27}$$

$$D\Omega_0 \, M_{3/2,-1/2} = 0 \tag{A28}$$

$$D\Omega_0 \, M_{-3/2,1/2} = 0 \tag{A29}$$

This agrees with our expectation that $U_{3/2-1/2R}$, $U_{-3/21/2R}$, $U_{1/2-1/2R}$ and $U_{-1/21/2R}$ do not have an angular momentum barrier. To get the effective Lagrangian inside the monopole core, we also need the effect of the Δ term.

$$\Delta L_{1/2,-1/2} = -2i\,(1-F(r))\,L_{-1/2,1/2} \tag{A30}$$

$$\Delta L_{-1/2,1/2} = 2i\,(1-F(r))\,L_{1/2,-1/2} \tag{A31}$$

$$\Delta M_{1/2,-1/2} = -2i\,(1-F(r))\,M_{-1/2,1/2} \tag{A32}$$

$$\Delta M_{-1/2,1/2} = 2i\,(1-F(r))\,M_{1/2,-1/2} \tag{A33}$$

$$\Delta M_{1/2,1/2} = \sqrt{3}\,(1-F(r))\,M_{3/2,-1/2} \tag{A34}$$

$$\Delta M_{-1/2,-1/2} = \sqrt{3}\,(1-F(r))\,M_{-3/2,1/2} \tag{A35}$$

$$\Delta M_{3/2,-1/2} = \sqrt{3}\,(1-F(r))\,M_{1/2,1/2} \tag{A36}$$

$$\Delta M_{-3/2,1/2} = \sqrt{3}\,(1-F(r))\,M_{-1/2,-1/2} \tag{A37}$$

Putting all these terms together, we get the effective Lagrangian 3.5 in section III.

APPENDIX B

In this appendix, we shall derive the boundary conditions that the fermions in the T = 3/2 representation have to satisfy in the presence of the classical monopole field, in the limit that the core radius is vanishingly small.

The equations of motion are obtained from the effective Lagrangian 3.5 and for convenience, we shall use the variables

$$A = U^{m}_{3/2,-1/2\,R} - U^{m}_{-3/2,1/2\,R} \qquad B1$$

$$B = U^{m}_{+3/2,-1/2\,R} + U^{m}_{-3/2,1/2\,R} \qquad B2$$

$$C = U^{m}_{1/2,+1/2\,R} - U^{m}_{-1/2,-1/2\,R} \qquad B3$$

$$D = U^{m}_{1/2,+1/2\,R} + U^{m}_{-1/2,-1/2\,R} \qquad B4$$

$$E = U^{m}_{1/2,-1/2\,R} - U^{m}_{-1/2,1/2\,R} \qquad B5$$

$$K = U^{m}_{1/2,-1/2\,R} + U^{m}_{-1/2,1/2\,R} \qquad B6$$

$$G = U^{J=0}_{1/2,-1/2\,R} - U^{J=0}_{-1/2,1/2\,R} \qquad B7$$

$$H = U^{J=0}_{1/2,-1/2\,R} + U^{J=0}_{-1/2,1/2\,R} \qquad B8$$

Since we are interested in the behaviour of the solutions near the monopole - i.e. when $r \ll E^{-1}$, - we may set $i\partial_0 U = EU \cong 0$. In this region, the equations of motion are

$$\partial_r A + \frac{\sqrt{3}\,i}{r}\left(1 - F(r)\right) D = 0 \qquad B9$$

$$\partial_r B + \frac{\sqrt{3}\,i}{r}\left(1 - F(r)\right) C = 0 \qquad B10$$

$$-\partial_r C + \frac{\sqrt{3}\,i}{r}\left(1-F(r)\right)B - \frac{\sqrt{2}\,i}{r}K = 0 \qquad B11$$

$$-\partial_r D + \frac{\sqrt{3}\,i}{r}\left(1-F(r)\right)A - \frac{\sqrt{2}\,i}{r}E = 0 \qquad B12$$

$$\partial_r E - \frac{\sqrt{2}\,i}{r}D - \frac{2}{r}\left(1-F(r)\right)E = 0 \qquad B13$$

$$\partial_r K - \frac{\sqrt{2}\,i}{r}C + \frac{2}{r}\left(1-F(r)\right)K = 0 \qquad B14$$

$$\partial_r G - \frac{2}{r}\left(1-F(r)\right)G = 0 \qquad B15$$

$$\partial_r H + \frac{2}{r}\left(1-F(r)\right)H = 0 \qquad B16$$

Equations B15 and B16 describe the motion of the J = 0 component and for finiteness of the solution at r = 0, we have to set

$$H(r_0) = 0 \qquad B17$$

This is exactly analogous to the T = 1/2 case.

For the J = 1 components, some unscrambling of the equations has to be done. Equations B9, B12 and B13 form one set, and B10, B11 and B14 form another set. We shall derive the boundary conditions under the following approximation

$$\left.\begin{array}{l} F(r) = 0,\ r<r_0 \\ F(r) = 1,\ r>r_0 \end{array}\right\} \qquad B18$$

but as we shall show later, the boundary conditions are valid for arbitrary F(r).

Let us solve the system of equations B9, B12 and B13 outside the monopole core. A decouples from D and E and equation B9 has the solution

$$A = \text{constant}, \quad r_0 < r \ll E^{-1} \qquad B19$$

B12 and B13 are coupled equations that can be solved yielding

$$D = d_1 r^{\sqrt{2}} + \frac{d_2 (r_0)^{\sqrt{2}-1/2}}{r^{\sqrt{2}}}, \quad r_0 < r \ll E^{-1} \qquad B20$$

$$E = i d_1 r^{\sqrt{2}} - \frac{i d_2 (r_0)^{\sqrt{2}-1/2}}{r^{\sqrt{2}}}, \quad r_0 < r \ll E^{-1} \qquad B21$$

where d_1 and d_2 are constants independent of r and r_0. The factors of r_0 in the numerator come from the requirement of square integrability of the solutions - i.e. $\int_{r_0}^{E^{-1}} |D|^2 dr$, $\int_{r_0}^{E^{-1}} |E|^2 dr$ = finite. These solutions imply the condition

$$iD + E = 0, \quad r = r_0 \qquad B22$$

Let us now consider the equations inside the monopole core. Eliminating A and E from the set of equations, we get the following third order differential equation for D.

$$r^2 \partial^3 D + r \partial^2 A - 6 \partial A + \frac{6A}{r} = 0 \qquad B23$$

with the solution

$$D = d_1 r + d_2 r^3 + \frac{d_3}{r^2}, \quad r < r_0 \qquad B24$$

The condition of square integrability ($\int_0^{r_0} |D|^2 dr$ = finite) eliminates the solution $1/r^2$, so that we are left with

$$D = d_1 r + d_2 r^3 \quad , \quad r < r_0 \qquad B25$$

From this, we also obtain

$$A = -\sqrt{3}\, i\, d_1 r - \frac{i}{\sqrt{3}} d_2 r^3 \quad , \quad r < r_0 \qquad B26$$

$$E = -\sqrt{2}\, i\, d_1 r + \sqrt{2}\, i\, d_2 r^3 \quad , \quad r < r_0 \qquad B27$$

This gives us the relation that

$$2iD + \sqrt{3} A - \frac{1}{\sqrt{2}} E = 0 \qquad , \quad r = r_0 \qquad B28$$

We carry out the same manipulations for the other set of equations B10, B11 and B14. From the solutions for $r > r_0$, by we get

$$iC + K = 0, \; r = r_0 \qquad B29$$

For $r < r_0$, by eliminating B and K, we get

$$r^2 \partial^3 C + 5 r \partial^2 C - 2 \partial C - \frac{6C}{r} = 0 \qquad B30$$

Which has the solution

$$C = k_1 r^2 + \frac{k_2}{r} + \frac{k_3}{r^3} \quad , \quad r < r_0 \qquad B31$$

Here the condition of square integrability leaves us with

$$C = k_1 r^2 \quad , \quad r < r_0 \qquad B32$$

The solutions for B and K are

$$B = -\sqrt{3}\, i\, k_1 r^2 \quad , \quad r < r_0 \qquad B33$$

$$K = \frac{i k_1}{2\sqrt{2}} r^2 \quad , \quad r < r_0 \qquad B34$$

These solutions lead to the following relations at $r = r_0$

$$\sqrt{3}\, i\, C + B = 0 \quad , \quad r = r_0 \qquad B35$$

$$\frac{-i}{2\sqrt{2}} C + K = 0 \quad , \quad r = r_0 \qquad B36$$

Now, we have conditions at $r = r_0$, obtained from solutions for $r > r_0$ and $r < r_0$, which have to match smoothly. For the set A,D and E, equations B22 and B28 can be satisfied with non-zero values of A,D and E at $r = r_0$. But for the set B,C and K, equations B29, B35 and B36 have the solutions

$$C(r_0) = 0 \qquad B37$$

$$K(r_0) = 0 \qquad B38$$

$$B(r_0) = 0 \qquad B39$$

Let us look at the effect of D and E for $r >> r_0$. The second term in equations B20 and B21 is negligible for this region.Hence,effectively, we may set $d_2 = 0$, when we are considering its effect for $r >> r_0$. In the limit, r_0 0, the first term is negligible in both the solutions. So,effectively, we may set

$$D(r_0) = 0 \qquad B40$$

$$E(r_0) = 0 \qquad B41$$

But, even though A,D and E are related at $r = r_0$, these effective boundary conditions cannot be used to set any boundary condition on A. Thus, we get the effective boundary conditions that we mentioned in section II. Though the boundary conditions have been derived using the step function approximation for $F(r)$, they are valid for arbitrary $F(r)$. Since $F(0) = 0$, the differential equations B23 and B30 do not change, so that we still have the same r dependence near the origin. Since $F(r)$ describes their evolution from 0 to r_0, different linear combinations may be zero at r_0 - i.e., we expect equations B28, B35 and B36 to change. But this does not change any of the boundary conditions.

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